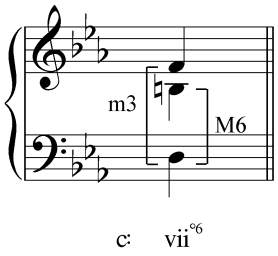
Lesson PPP: Fully-Diminished Seventh Chords

Introduction:

In Lesson 6 we looked at the diminished leading-tone triad: vii^{o}. There, we discussed why the tritone between the root and fifth of the chord requires special attention. The chord usually appears in first inversion precisely to avoid that dissonant interval sounding against the bass when vii^{o} is in root position.

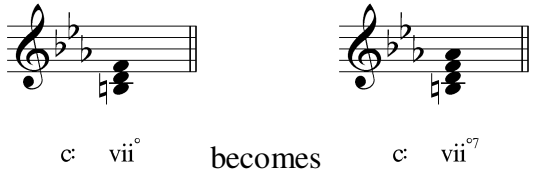
Example 1:



As Example 1 demonstrates, placing the chord in first inversion ensures that the upper voices are consonant with the bass. The diminished fifth is between the alto and soprano, concealed within the upper voices. In this case, it is best understood as a resultant interval formed as a result of avoiding dissonances involving the bass.

Adding a diatonic seventh to a diminished leading-tone triad in minor will result in the following sonority:

Example 2:



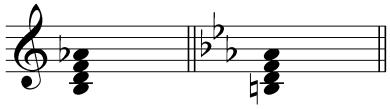
This chord consists of a diminished triad with a diminished seventh added above the root. It is therefore referred to as a fully-diminished seventh chord.

In this lesson, we will discuss the construction of fully-diminished seventh chords in major and minor keys. As you will see, the chord consists of two interlocking tritones, which require particularly careful treatment because of their strong voice-leading tendencies. We will consider its various common functions and will touch on several advanced uses of the chord as well.

Construction:

Fully-diminished leading-tone seventh chords can be built in major or minor keys. In Roman numeral analyses, they are indicated with a degree sign followed by seventh-chord figured bass numerals, depending on inversion (^{o7}, $^{o\,6}_{5}$, $^{o\,4}_{3}$, or $^{o\,4}_{2}$). The following example shows the construction of the chord in C major and in C minor:

Example 3:



C: vii^{o7} c: vii^{o7}

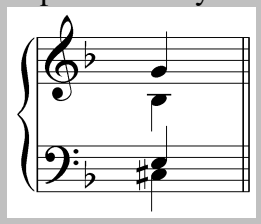
Fully-diminished seventh chords cannot be constructed from only diatonic notes. In other words—as Example 3 demonstrates—a mixture tone is always required. Fully-diminished seventh chords in major borrow ♭6̂ from the parallel minor, while those in minor borrow the leading tone from the parallel major.

Activity PPP.01:

Build fully-diminished seventh chords on the leading tone of each of the following keys. (Remember to use ♭6̂ in major keys and to raise the leading tone in minor.)

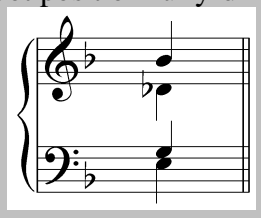
Exercise PPP.01a

Build a root position fully-diminished leading tone seventh chord in D minor.

[Answer:  (answers may vary, provided C♯ is in the bass with E, G, B♭ in the upper voices). Response if correct: "Correct!" Response if C instead of C♯ but otherwise correct: "Almost. Remember to raise the leading tone in minor." Response if incorrect: "Incorrect. Try again."]

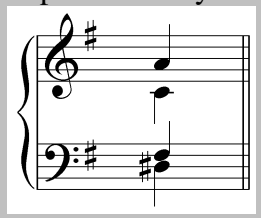
Exercise PPP.01b

Build a root position fully-diminished leading tone seventh chord in F major.

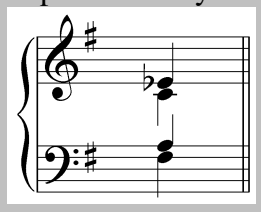
[Answer:  (answers may vary, provided E is in the bass with G, B♭, D♭ in the upper voices). Response if correct: "Correct!" Response if D instead of D♭ but otherwise correct: "Almost. Remember that vii^{o7} uses ♭6̂ in major keys." Response if incorrect: "Incorrect. Try again."]

Exercise PPP.01c

Build a root position fully-diminished leading tone seventh chord in E minor.

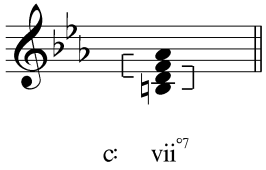
[Answer:  (answers may vary, provided D♯ is in the bass with F♯, A, C in the upper voices). Response if correct: "Correct!" Response if D instead of D♯ but otherwise correct: "Almost. Remember to raise the leading tone in minor." Response if incorrect: "Incorrect. Try again."]

Exercise PPP.01d
Build a root position fully-diminished leading tone seventh chord in G major.

[Answer:  (answers may vary, provided F♯ is in the bass with A, C, E♭ in the upper voices). Response if correct: "Correct!" Response if E instead of E♭ but otherwise correct: "Almost. Remember that vii^{o7} uses ♭6̂ in major keys." Response if incorrect: "Incorrect. Try again."]

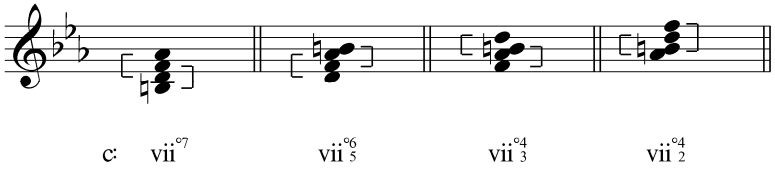
The dissonant sound of a fully-diminished seventh chord is striking. The combination of a diminished triad with a diminished seventh above the root yields two interlocking tritones. The brackets in the following example indicate the two inherent tritones of a fully-diminished seventh chord:

Example 4:



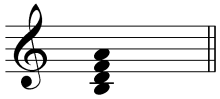
As you can see, every member of the chord forms a tritone with some other member. (Note: Historically, "tritone" describes augmented fourths, which are composed of three whole tones. Here, we will use the term generically to refer to augmented fourths *and* diminished fifths, their enharmonic equivalent.) The result is that the bass is always in a tritone-forming relationship with some other voice (unlike the vii^{o6} chord, where the single tritone can be hidden between upper voices).

Example 5:



Note: The prefix "fully-" in the name "fully-diminished seventh chord" refers to the fact that the chord is constructed of a diminished triad *and* a diminished seventh. But consider the diatonic leading-tone seventh chord in major:

Example 6:



Without ♭6̂, as in Example 3, the chord contains a minor seventh above the root instead of a diminished seventh. This sonority is referred to as a *half*-diminished seventh chord and is indicated in Roman numeral analyses by a degree sign with a slash through it (vii^{ø7}). Half-diminished seventh chords occur somewhat less frequently than their fully-diminished counterparts, and rarely in minor keys. They are typically found as neighboring auxiliary sonorities expanding a I chord.

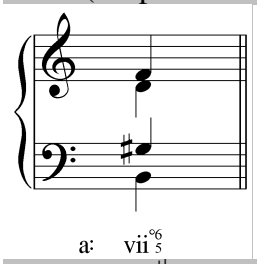
Fully-diminished seventh chords have a unique effect that composers exploit in a number of ways. Primarily, however, they function as dominant substitutes.

Activity PPP.02:

Identify the diminished fifths and augmented fourths in the following inverted fully-diminished seventh chords.

Exercise PPP.02a

Identify the diminished fifth between the root and fifth of the following fully-diminished seventh chord (keep in mind it may appear in inversion as an augmented fourth):



[Answer: G♯ and D. Response if correct: "Correct!" Response if incorrect: "Incorrect. Try again."]

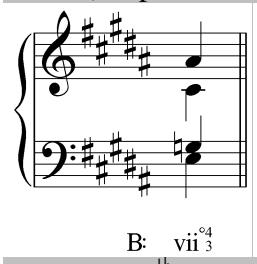
[Follow-up question:]

Now identify the remaining diminished fifth between the third and seventh of the chord.

[Answer: B and F. Response if correct: "Correct!" Response if incorrect: "Incorrect. Try again."]

Exercise PPP.02b

Identify the diminished fifth between the root and fifth of the following fully-diminished seventh chord (keep in mind it may appear in inversion as an augmented fourth):



[Answer: A♯ and E. Response if correct: "Correct!" Response if incorrect: "Incorrect. Try again."]

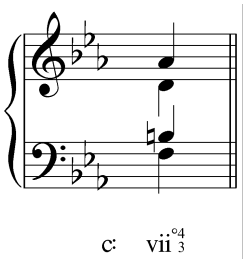
[Follow-up question:]

Now identify the remaining diminished fifth between the third and seventh of the chord.

[Answer: C♯ and G. Response if correct: "Correct!" Response if incorrect: "Incorrect. Try again."]

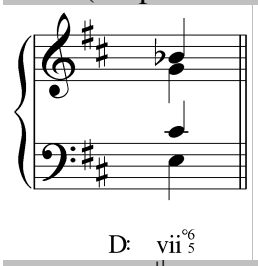
Exercise PPP.02c

Identify the diminished fifth between the root and fifth of the following fully-diminished seventh chord (keep in mind it may appear in inversion as an augmented fourth):



[Answer: B natural and F. Response if correct: “Correct!” Response if incorrect: “Incorrect. Try again.”]
[Follow-up question:]
Now identify the remaining diminished fifth between the third and seventh of the chord.
[Answer: D and A♭. Response if correct: “Correct!” Response if incorrect: “Incorrect. Try again.”]

Exercise PPP.02d
Identify the diminished fifth between the root and fifth of the following fully-diminished seventh chord (keep in mind it may appear in inversion as an augmented fourth):

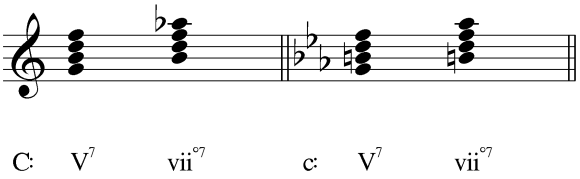


[Answer: C♯ and G. Response if correct: “Correct!” Response if incorrect: “Incorrect. Try again.”]
[Follow-up question:]
Now identify the remaining diminished fifth between the third and seventh of the chord.
[Answer: E and B♭. Response if correct: “Correct!” Response if incorrect: “Incorrect. Try again.”]

Function and resolution:

Consider the following comparison of dominant seventh chords and leading-tone seventh chords:

Example 7:

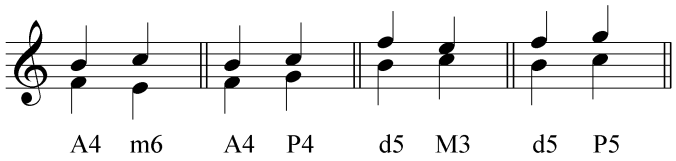


As you can see from Example 7, fully-diminished seventh chords built on the leading tone have three pitches in common with dominant seventh chords (scale degrees $\hat{7}$, $\hat{2}$, and $\hat{4}$). The only difference is that they include $\flat\hat{6}$ (diatonic $\hat{6}$ in minor) instead of scale degree $\hat{5}$. Because they share three chord members, fully-diminished leading-tone seventh chords typically function as dominant substitutes.

Resolving a fully-diminished seventh chord requires careful handling of the tritones. As mentioned above, one of them inevitably involves the bass. As such, it tends to stand out and must be treated with care.

In Lesson 6 we expanded the list of basic interval progressions (see Lesson 1) to accommodate chords that include a tritone. There, we discussed several possible resolutions. Example 8 summarizes:

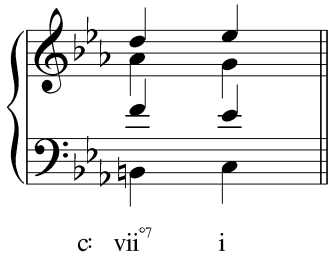
Example 8:



Typically, the tritones in fully-diminished seventh chords resolve by contrary or similar motion: augmented fourths resolve outward to sixths or in similar motion up to perfect fourths while diminished fifths resolve inward to thirds or in similar motion up to a perfect fifth.

Consider the following example where vii^{o7} resolves to i in C minor:

Example 9:

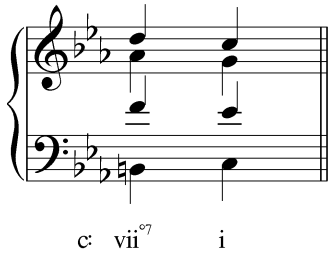


In Example 9, the bass (B natural) forms a diminished fifth with the tenor (F). As vii^{o7} resolves to I, we can see this tritone contracting to a minor third (C and E♭). Likewise, the augmented fourth between the alto and soprano (A♭ and D) expands to form a minor sixth (G and E♭). Note the resulting doubled third in the tonic chord, which is common after fully-diminished leading tone chords. Typically, contrary motion of this sort is the favored method of resolving tritones. Composer will occasionally resolve a tritone using similar motion, but will frequently restrict such an interval progression to the upper voices.

Most of the tones in a fully-diminished leading tone chord have a strong tendency to resolve to the pitches of a tonic triad. The leading tone is pulled towards the tonic. The seventh of the chord, ♭$\hat{6}$, resolves like any other seventh: down by step (in this case to $\hat{5}$). (Refer to Lesson 7 for more information on seventh chords.) And finally, $\hat{4}$ is drawn downward to $\hat{3}$. For these reasons, it may be helpful to think about the resolution of a fully-diminished seventh chord in terms of its tendency tones.

Look again at Example 9. Each of the tendency tones resolves as expected: the leading tone steps up to the tonic in the bass while $\hat{6}$ steps down to $\hat{5}$ in the alto and $\hat{4}$ steps down to $\hat{3}$ in the tenor. The remaining voice, scale degree $\hat{2}$, can move to either $\hat{3}$—as it does in Example 9—or to the tonic, as in the following example:

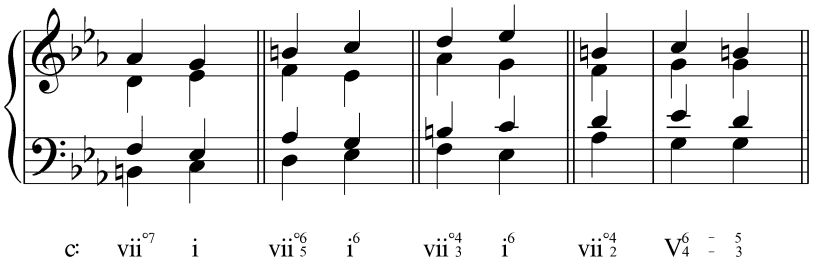
Example 10:



Note that in Example 10 the augmented fourth formed by the alto and soprano now resolves with similar motion to a perfect fourth.

Fully-diminished seventh chords can appear in any position:

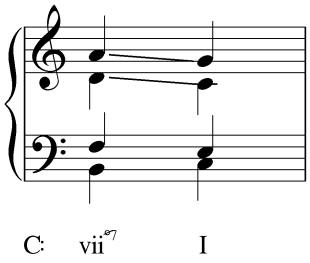
Example 11:



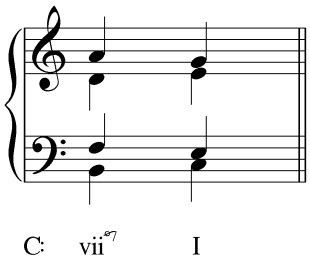
Third-inversion fully-diminished seventh chords are less common than the other positions. With scale degree $\hat{6}$ in the bass ($\flat\hat{6}$ in major), the chord tends to resolve to a cadential $^{6}_{4}$ chord or an auxiliary I^{6}_{4}. Note that in the resolution of $\mathrm{vii}^{o}{}^{4}_{2}$ in Example 11 scale degree $\hat{4}$ in the alto voice steps up to $\hat{5}$, resolving the augmented fourth in similar motion to a perfect fourth.

Note: Half-diminished seventh chords contain only one tritone. The fifth between scale degrees $\hat{2}$ and $\hat{6}$ in major is perfect. The resolution of a half-diminished seventh chord requires extra attention to avoid parallel fifths:

Example 12:



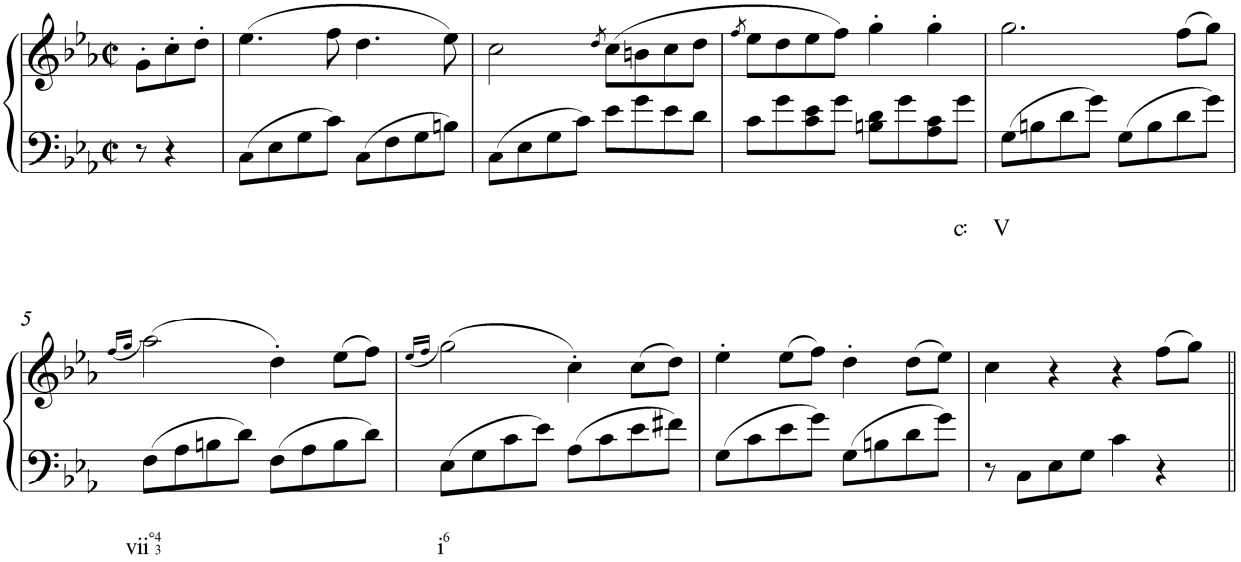
Example 13:



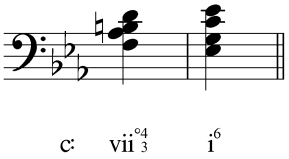
In Example 12, the alto and soprano form parallel fifths. This can be avoided by doubling the third of the resolution chord, as in Example 13.

Now consider the following example from Beethoven's famous "Pathétique" sonata (Example 15 provides a reduction of mm. 5-6):

Example 14 (L. Beethoven, Sonata no. 8 ("Pathétique"), Op. 13, Mvt. I, mm. 1-8):



Example 15:



In m. 5 we encounter a fully-diminished seventh chord in second inversion. Looking at the left-hand part, we see that all of the voices resolve as expected. Both of the tritones appear as augmented fourths and expand outward by contrary motion to sixths: F and B natural move in contrary motion to E♭ and C while A♭ and D do the same, to G and E♭.

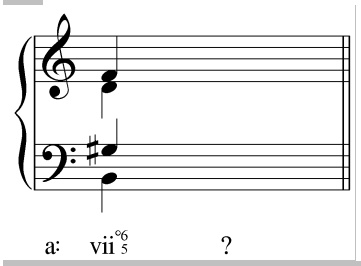
Activity PPP.03:

Resolve the following fully-diminished seventh chords according to the voiceleading procedures outlined above.

Exercise PPP.03a

[Multiple choice question:]

To what chord would the following fully-diminished seventh in first inversion normally resolve to?

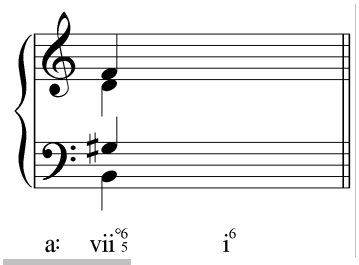


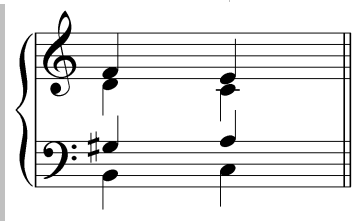
[Options: "i," "i6," "cadential 6/4 or auxiliary 6/4"]

[Answer: i6. Response if correct: "Correct!" Response if incorrect: "Incorrect. (Hint: Which chord member is in the bass? To where does this pitch normally resolve?)"]

[Follow-up question:]

Resolve the fully-diminished seventh chord:



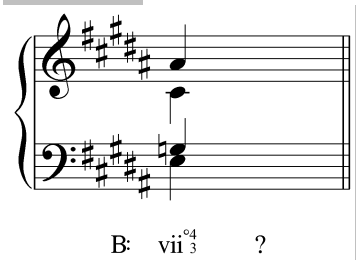
[Answer: 

. Response if correct: "Correct!" Response if incorrect: "Incorrect. Remember to resolve all tendency tones in the usual manner."]

Exercise PPP.03b

[Multiple choice question:]

To what chord would the following fully-diminished seventh in second inversion normally resolve to?

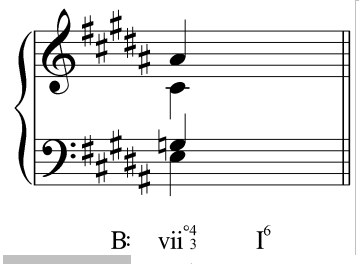


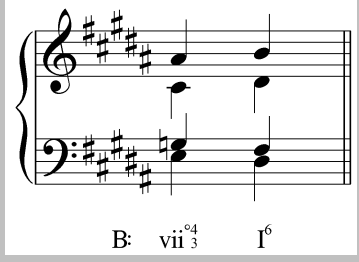
[Options: "i," "i^6," "cadential $\frac{6}{4}$ or auxiliary $\frac{6}{4}$"]

[Answer: i^6. Response if correct: "Correct!" Response if incorrect: "Incorrect. (Hint: Which chord member is in the bass? To where does this pitch normally resolve?)"]

[Follow-up question:]

Resolve the fully-diminished seventh chord:

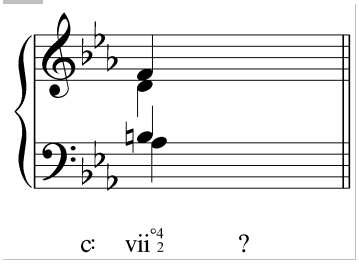


[Answer: 

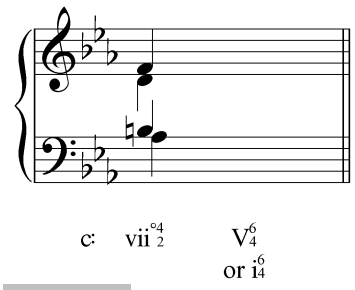
(alternatively, C♯ in the alto may resolve to B). Response if correct: "Correct!" Response if incorrect: "Incorrect. Remember to resolve all tendency tones in the usual manner."]

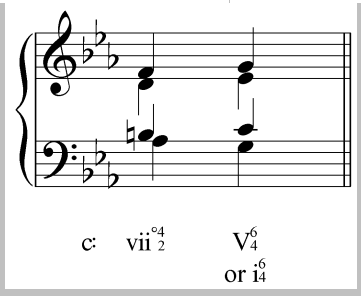
Exercise PPP.03c

[Multiple choice question:]
To what chord would the following fully-diminished seventh in third inversion normally resolve to?



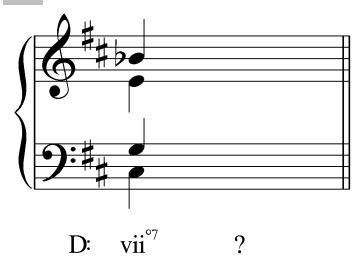
[Options: "i," "i^6," "cadential $\frac{6}{4}$ or auxiliary $\frac{6}{4}$"]
[Answer: cadential $\frac{6}{4}$ or auxiliary $\frac{6}{4}$. Response if correct: "Correct!" Response if incorrect: "Incorrect. (Hint: Which chord member is in the bass? To where does this pitch normally resolve?)"]
[Follow-up question:]
Resolve the fully-diminished seventh chord:



[Answer: 

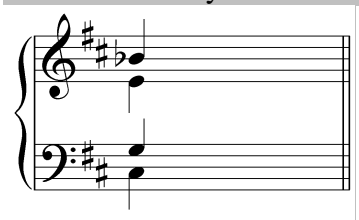
(alternatively, D in the alto may resolve to C). Response if correct: "Correct!" Response if incorrect: "Incorrect. Remember to resolve all tendency tones in the usual manner."]

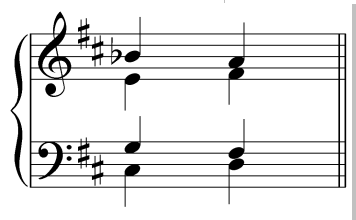
Exercise PPP.03d
[Multiple choice question:]
To what chord would the following fully-diminished seventh in root position normally resolve to?



[Options: "i," "i^6," "cadential $\frac{6}{4}$ or auxiliary $\frac{6}{4}$"]
[Answer: i. Response if correct: "Correct!" Response if incorrect: "Incorrect. (Hint: Which chord member is in the bass? To where does this pitch normally resolve?)"]

[Follow-up question:]
Resolve the fully-diminished seventh chord:



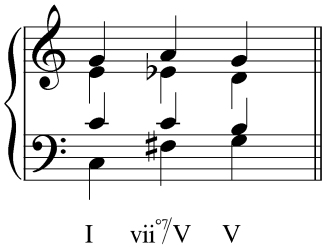


[Answer: (alternatively, E in the alto may resolve to D). Response if correct: "Correct!" Response if incorrect: "Incorrect. Remember to resolve all tendency tones in the usual manner."]

As applied or pivot chords:

In Lesson 10, we looked at applied chords. We discussed how fully-diminished leading tone seventh chords, like dominant seventh chords, can be used to tonicize triads other than the tonic. The following example shows a tonicization of V with an applied fully-diminished chord:

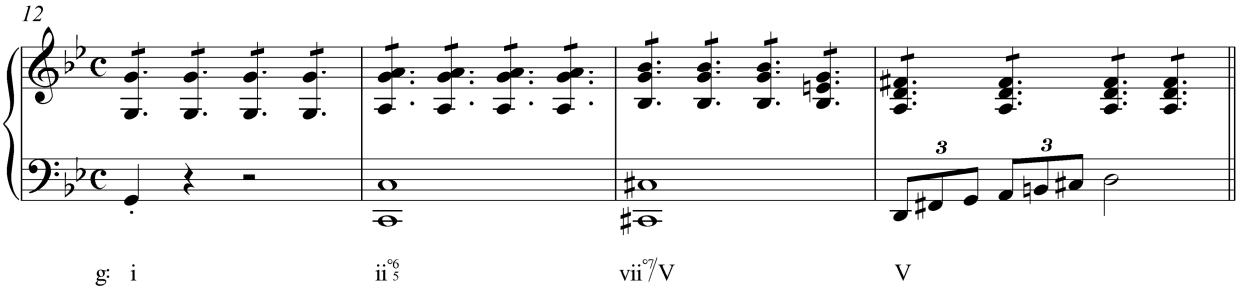
Example 16:



Note that while it would typically be permissible for the augmented fourth between the alto and soprano to resolve outward to a major sixth, here the soprano must step down in order to avoid doubling the leading tone in V.

Now consider the following example:

Example 17 (F. Schubert, "Erlkönig," D 328, mm. 12-15):

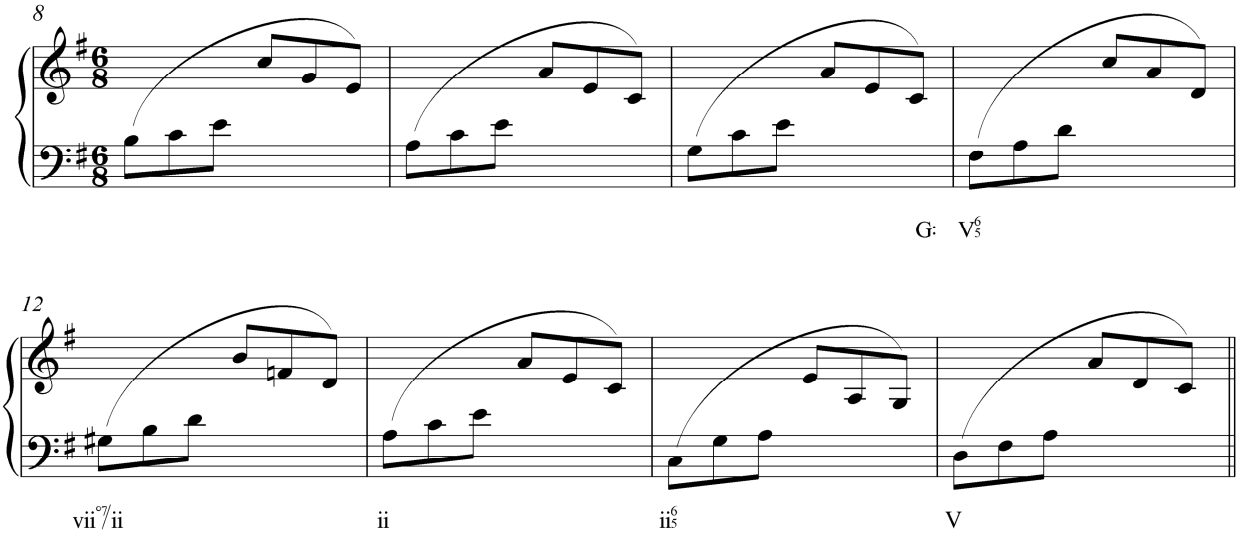


In m. 14 of this Schubert song we find an applied fully-diminished chord tonicizing V. As you can see, all of the usual considerations for resolving a fully-diminished chord remain the same. The diminished

fifth formed by C♯ and G contracts inward to a major third, while the augmented fourth between B♭ and E natural moves in similar motion to a perfect fourth.

Other chords can be tonicized in this way as well. In the following example, ii is tonicized with a fully-diminished chord in mm. 12-13:

Example 18 (R. Schumann, Album for the Young, No. 14: "Kleine Studie," mm. 8-15):

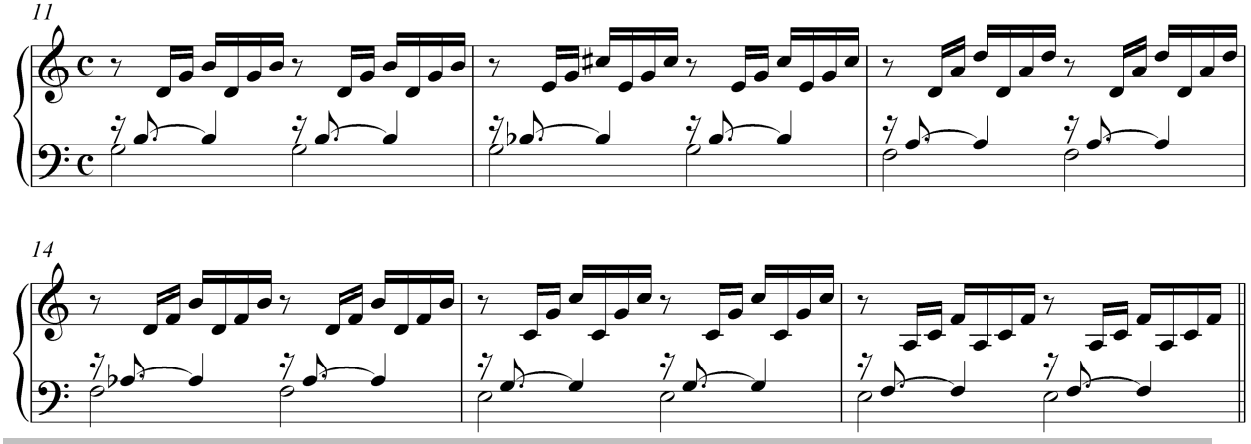


Activity PPP.04:

Identify the applied fully-diminished seventh chords in each of the following excerpts and indicate the chords they are tonicizing.

Exercise PPP.04a

The following excerpt in C major (J.S. Bach, Prelude and Fugue 1 in C major (from *The Well-Tempered Clavier*, Book I), mm. 11-16) contains an applied fully-diminished seventh chord. Click on it:



[Answer: all of m. 12. Response if correct: "Correct!" Response if incorrect: "Incorrect. (Hint: Look for a temporary leading tone.)

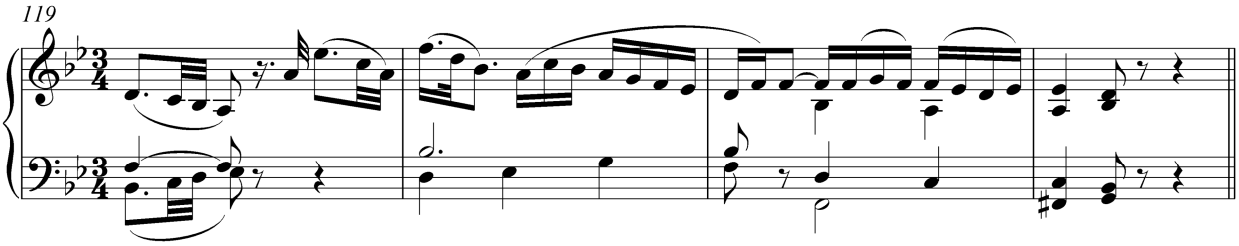
[Follow-up question:]

What chord does the applied fully-diminished seventh tonicize?

[Answer: ii. Response if correct: "Correct! The ii chord in m. 13 is tonicized." Response if incorrect: "Incorrect. (Hint: To which scale degree does the temporary leading tone—the root of the vii^{o7} chord—resolve?)"]

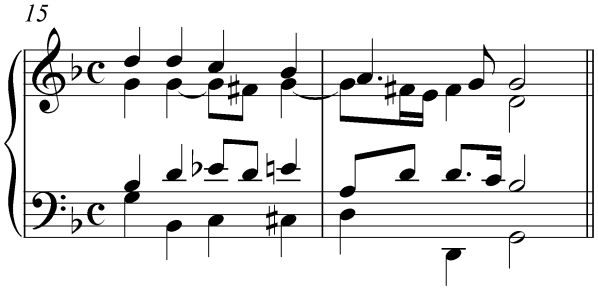
Exercise PPP.04b

The following excerpt in B♭ major (W.A. Mozart, Fantasia in C minor, K. 475, mm. 119-122) contains an applied fully-diminished seventh chord. Click on it:



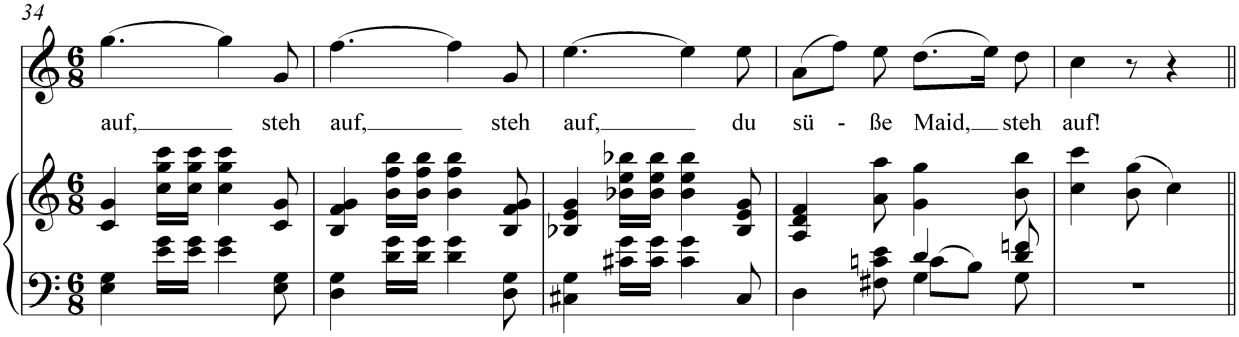
[Answer: first beat of m. 122. Response if correct: "Correct!" Response if incorrect: "Incorrect. (Hint: Look for a temporary leading tone.)
[Follow-up question:]
What chord does the applied fully-diminished seventh tonicize?
[Answer: vi. Response if correct: "Correct! The applied chord tonicizes vi, emphasizing the deceptive cadence." Response if incorrect: "Incorrect. (Hint: To which scale degree does the temporary leading tone—the root of the vii^{o7} chord—resolve?)"]

Exercise PPP.04c
The following excerpt in G minor (J.S. Bach, "Was betrübst du dich, mein Herze" (BWV 423), mm. 15-16) contains an applied fully-diminished seventh chord. Click on it:



[Answer: fourth beat of m. 15. Response if correct: "Correct!" Response if incorrect: "Incorrect. (Hint: Look for a temporary leading tone.)
[Follow-up question:]
What chord does the applied fully-diminished seventh tonicize?
[Answer: V. Response if correct: "Correct! The applied diminished seventh tonicizes the dominant harmony of m. 16." Response if incorrect: "Incorrect. (Hint: To which scale degree does the temporary leading tone—the root of the vii^{o7} chord—resolve?)"]

Exercise PPP.04d
The following excerpt in C major (F. Schubert, "Horch, horch! Die Lerch," D. 889, mm. 34-38) contains an applied fully-diminished seventh chord. Click on it:



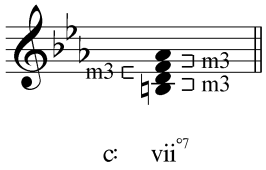
[Answer: all of m. 36. Response if correct: "Correct!" Response if student clicks on the third eighth note of m. 37: "Almost. That chord is an applied leading-tone chord, but is half-diminished." Response if incorrect: "Incorrect. (Hint: Look for a temporary leading tone.)

[Follow-up question:]
What chord does the applied fully-diminished seventh tonicize?
[Answer: ii. Response if correct: "Correct!" Response if incorrect: "Incorrect. (Hint: To which scale degree does the temporary leading tone—the root of the vii^{o7} chord—resolve?)"]

Fully-diminished seventh-chords can also be used as pivot chords in modulations. They are particularly useful in this regard when modulating to distant keys.

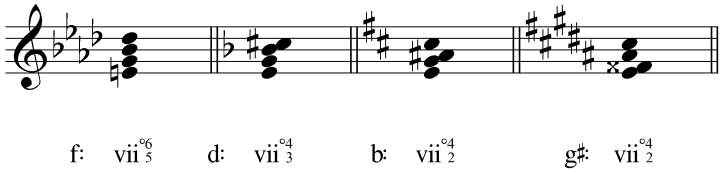
Consider the structure of a fully diminished seventh chord. Above, we described the sonority as a diminished triad with a diminished seventh added above the root. You can also think of it as a stack of minor thirds:

Example 19:



Stacking another minor third on top of this would result in the enharmonic equivalent of the root—in this case C♭, the enharmonic equivalent of B natural. The implication of this unique property is that any of the four pitches can be interpreted and heard as the root of an applied fully-diminished chord. The following example shows how the same chord can be enharmonically interpreted as vii^{o7} in four different keys:

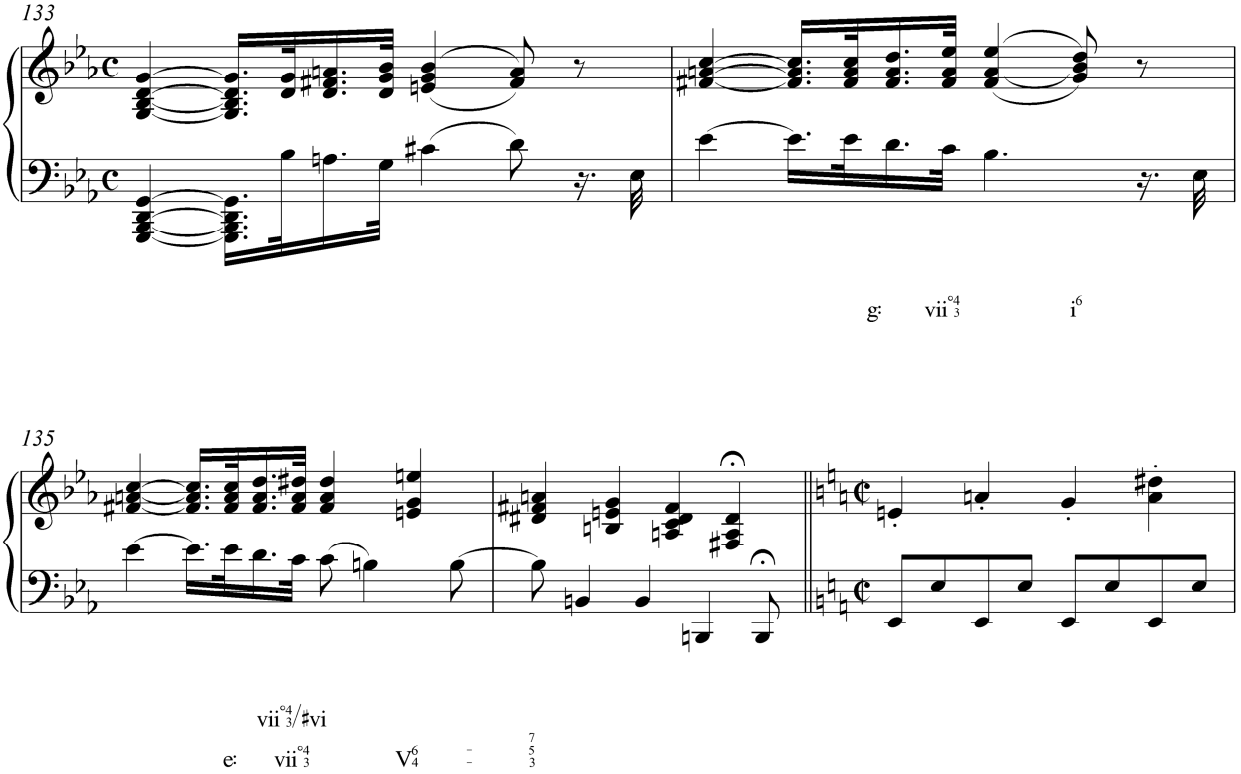
Example 20:



Each of the chords in Example 20 sounds exactly the same. Because of its special construction, a fully-diminished seventh chord can be heard in four different ways.

In both Examples 17 and 18, the unique, immediately, aurally identifiable quality of the fully-diminished chord facilitates tonicizations. Composers exploit this recognizable chord and its potential for enharmonic reinterpretation in chromatic modulations. Consider the following example:

Example 21 (L. Beethoven, Sonata no. 8 ("Pathétique"), Op. 13, Mvt. I, mm. 133-137):

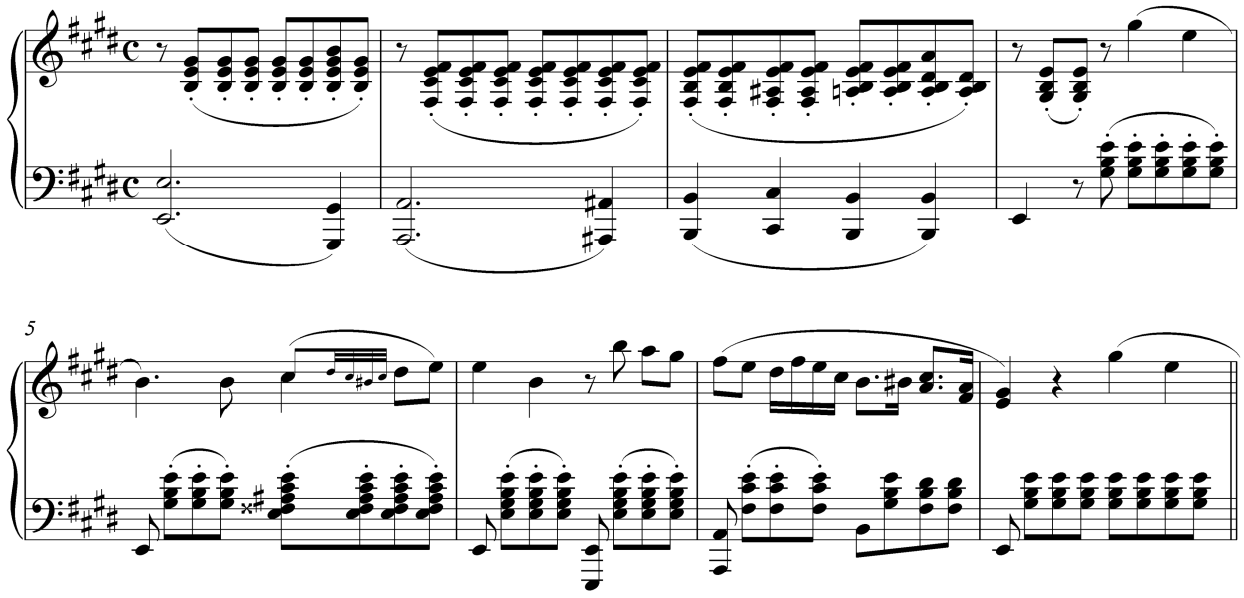


In Example 21, we first encounter a fully-diminished seventh chord in m. 134: vii$^{o\,4}_{\ 3}$ resolves to i^6 in G minor. In the next measure, however, E♭ (the seventh of vii^{o7}) is respelled as D♯. The altered notation signals a change in function. Instead of leading to the tonic, the fully-diminished chord now functions as an applied leading-tone chord to E minor (♯vi in G minor). By reinterpreting the seventh of the original chord as the root, Beethoven modulates smoothly from G minor to the distant key of E minor.

As auxiliary sonorities:

Fully-diminished seventh chords also appear as auxiliary sonorities, as in the following example:

Example 22 (F. Mendelssohn, Rondo Capriccioso, Op.14, mm 1-8):



In m. 5 of this piece by Mendelssohn, we find a chromatic sonority consisting of F×, A♯, C♯, and E. At first, this fully-diminished seventh may appear to be an applied chord.

Activity PPP.05:

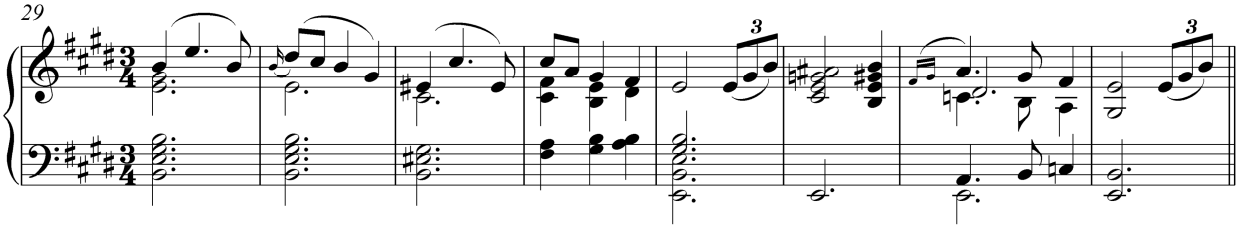
If this fully-diminished chord *were* an applied chord (vii^{o7}/?), what is the Roman numeral of the chord it would be tonicizing?]

[Answer: iii. Response if correct: "Correct! This sonority resembles an applied chord tonicizing iii, but moves to I instead." Response if incorrect: "Incorrect. (Hint: What is the root of the chord as notated? That pitch could serve as the temporary leading tone to which scale degree?)"]

The chord is flanked on either side by tonic triads, indicating that it is not an applied chord, but a neighboring auxiliary sonority. E is sustained throughout, while ♯$\hat{2}$ (F×) and ♯$\hat{4}$ (A♯) are chromatic neighbors to $\hat{3}$ and $\hat{5}$, and $\hat{6}$ (C♯) is a diatonic neighbor of $\hat{5}$. Because it shares a pitch with the reference chord, this type of auxiliary sonority is widely referred to as a common-tone diminished seventh.

Activity PPP.06:

The following excerpt (F. Schubert, Moment Musical no. 6, Op. 94, D. 780, mm. 29-36) contains an auxiliary fully-diminished seventh. Click on it:



[Answer: all of m. 34. Response if correct: "Correct!" Response if incorrect: "Incorrect. Try again."]

[Follow-up question:]

This fully-diminished seventh is a neighboring auxiliary sonority. What harmony does it expand? (Note, that the excerpt is in E major.)

[Answer: I. Response if correct: “Correct! The sonority expands the phrase-ending tonic harmony.” Response if incorrect: “Incorrect. (Hint: What harmonies appear before and after the fully-diminished seventh?)”]

Conclusion:

Fully-diminished seventh chords are built by adding a diminished seventh to a diminished triad. They inherently contain two interlocking tritones (which may appear as diminished fifths or augmented fourths) which must be resolved carefully, particularly since one of them is inevitably emphasized by the bass. Typically, composers resolve the tritones by contrary motion: augmented fourths expanding outward to sixths, diminished fifths contracting inward to thirds. Occasionally, one of the tritones will resolve in similar motion to a perfect fourth or fifth, but that voiceleading is usually restricted to upper voices.

These chords, when built on the leading tone, include three strong tendency tones leading to pitches of the tonic triad. $\hat{7}$ is pulled upward to $\hat{1}$ while $\hat{4}$ and $\hat{6}$ are pulled down to $\hat{3}$ and $\hat{5}$, respectively. $\hat{2}$ resolves either to $\hat{1}$ or $\hat{3}$ depending on context.

While the chord can appear in any position, third-inversion fully-diminished chords are rare. With $\flat\hat{6}$ in the bass, they tend to resolve to an auxiliary I^{6}_{4} or a cadential $^{6}_{4}$ chord.

In addition to being used as auxiliary sonorities, fully-diminished seventh chords also appear as applied chords or as pivots in chromatic modulations. Their structure—evenly dividing the octave into minor thirds—makes it possible to interpret them enharmonically in four different keys: any of the four members can be heard as the root. That special property makes them particularly useful in modulations to distantly related keys.